One-Cycle Controlled Three-Phase PWM Rectifiers with Improved Regulation under Unbalanced and Distorted Input Voltage Conditions

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Abstract—This paper proposes a modified one-cycle-control (OCC) scheme for regulation of three-phase pulsewidth-modulated (PWM) rectifiers under unbalanced and distorted supply conditions. Mathematical analysis is presented to show the dominant harmonic voltage across the dc-link of PWM rectifier when controlled by the existing OCC scheme. Such ripple voltage may aggravate the harmonic distortion in input currents. To address these problems, the concept concerning with the reconstruction of reference signals in OCC core is introduced. With minor modifications made into the OCC control equation, the overall proposed control scheme can effectively minimize the harmonic components found in the input currents and smoothen output ripple imposed on the dc-link voltage without using large capacitance. Furthermore, previously identified advantages of OCC can all be preserved by the new control scheme. Finally, all the theoretical findings are supported by the experimental results obtained from a 1.2 kW three-phase PWM rectifier developed in the laboratory.

Keywords—PWM rectifier; harmonic compensation; unbalanced network; one-cycle-control

I. INTRODUCTION

Nowadays, the increasing usage of diode and thyristor based rectifiers results in serious harmonic problems in the utility grid. As an attractive alternative, the boost type PWM rectifier, which can provide high input power factor, nearly sinusoidal input currents, and maintain constant dc-link voltage with small capacitance, has drawn a lot of attentions and been widely employed for high-performance ac-dc conversion applications over the past few years. Nevertheless, all these features can only be obtained under ideal input voltage conditions. When the input voltages are unbalanced or contain harmonics, the performance of PWM rectifiers may degrade significantly if the applied control scheme is not properly designed.

Extensive research works have been carried out to investigate the operation of PWM rectifiers under distorted and unbalanced supply voltages. In [1], a mathematical model was established in the abc frame, and it was shown that the negative-sequence component in the unbalanced network can cause even order harmonics in the dc-link voltage, which in turn result in non-zero-sequence third-order harmonics in the input currents. However, the solution provided in the paper is to adopt bulky filter inductors and dc-link capacitor for harmonic suppression, this will no doubt increase the cost of the system. Based on the instantaneous active and reactive power control, [2] proposed a simple source voltage sensor-less direct power control (DPC), which can provide satisfactory results with slightly distorted supply voltage, but the unbalanced case is not studied and the application of this control method is limited due by the high sampling frequency required. To better understand the impact of unbalanced input voltages on the output dc-link voltage, [3] derived a generalized model in the dq synchronous frame and it is also proven that the dc-link voltage can be kept as harmonic free as long as a set of 4 by 4 matrix control equation is satisfied. Based on similar control theory, [4] developed a dual frame controller to regulate the positive- and negative-sequence individually and minimize the variations on the dc-link voltage. Although the total harmonic distortion (THD) in the input currents can be reduced to an acceptable extent, the three-phase currents are still unbalanced. Moreover, this control equation doesn’t hold any more when the source voltages are distorted. In this case, the harmonics originated from the supply voltages may pollute the input current even if a constant dc-link voltage is maintained by proper closed-loop control. Recently, [5] proposed a multiple reference frame-based control scheme to handle the unbalanced and distorted input conditions simultaneously. However, this method cannot completely eliminate the harmonic interference between different reference frames unless a more sophisticated decoupling technique is used. Reference [6] attempted to utilize the classical repetitive control, which can be commonly found in applications like active power filters (APF) and uninterruptible power supplies (UPS), to mitigate the periodic tracking errors for more precise current regulation. Again, its requirements of complex frame transformations, positive- and negative-sequence extraction, current reference calculation and dual inner loop regulation make the control algorithm very complicated, and therefore it might still not be the best option for many practical implementations.
In order to simplify the control algorithm, one-cycle control (OCC) has previously been introduced for the regulation of PWM rectifier [7, 8], after its smooth applications in DC-DC converters and power factor correction (PFC) circuits. Being different from most of the existing control schemes, OCC doesn’t require phase-lock loop (PLL) for current reference generation, and its nonlinear carrier characteristic permits a very fast response of the converter under load transient or external disturbances. Moreover, it can retain all the inherent merits of a PWM rectifier, such as unity power factor and power regeneration [8]. Unfortunately, this method also suffers from severe performance deterioration when the supply voltages are non-ideal. Referring to past literature, only a few papers have investigated the operation of OCC-based PWM rectifier under unbalanced and distorted input voltage conditions. In [9], it is proven that the line currents can be sinusoidal under unbalanced source and load conditions, however no compensation scheme was proposed for correcting the current unbalance concern, and a large capacitance is needed to hold the dc-link voltage constant so as to reduce the distortion on the supply side current.

In this paper, the concept concerning with the reconstruction of reference signals is introduced to address the harmonic problems faced on the ac input- and dc output-side of the PWM rectifiers. This technique can be conveniently embedded into the inner current loop of the conventional OCC. With minor modifications subsequently introduced, the overall proposed control scheme can effectively minimize the harmonic components found in the input currents and smoothen out ripple imposed on the dc-link voltage without using large capacitance. Additionally, previously identified advantages of OCC such as PLL-less, no frame transformation and constant switching frequency are all preserved by the new control scheme, whose details are now elaborated in the paper. All the theoretical findings are eventually verified via experimental testing on a 1.2 kW three-phase PWM rectifier developed in the laboratory.

II. BEHAVIOR OF OCC-BASED PWM RECTIFIER UNDER NONIDEAL SUPPLY VOLTAGES

Fig. 1 shows the circuit diagram of a typical boost type three-phase PWM rectifier. Notice that the LCL-filter is usually adopted in high power applications for better switching harmonic attenuation. Being similar with other control methods, OCC also employs a double loop control scheme. In the outer voltage loop, \( V_{dc} \) is regulated by a simple PI controller, whose output signal \( V_{m} \) reflects the load power level. The inner current loop is controlled on a cycle-by-cycle basis, and this nonlinear process can be described by the following control equation (derivation of (1) can be found in [7]),

\[
V_m(1 - 2d_x) = R_i i_{sx}, \quad (x = a, b, c)
\]  

where \( R_i \) is the equivalent current sensing resistance, \( i_{sx} \) represents the three-phase line currents and \( d_x \) is the duty ratio of \( S_a, S_b \) or \( S_c \). However, the above control equation may cause instability problem under light load conditions. A more practically orientated OCC can be realized by using a combination of source voltages and line currents for the inner current loop regulation as recommended in [8]. Then, the final control key equation for the modified OCC can be written as,

\[
V_m(1 - 2d_x) = R_i i_{sx} + K v_{sx}
\]

where \( v_{sx} \) is the instantaneous ac voltage, and \( K \) is a constant that should be selected to ensure a stable system even under no load condition, and its value is dependent on the adopted switching frequency, filter inductance and modulation index [8]. The realization of this control equation can be illustrated by the control diagram shown in Fig. 2. On the ac input side, the following equation can be readily established by ignoring the voltage drop across the filter inductors,

\[
v_{sx} = v_{cx} = V_{dc} S_x = V_{dc}(1 - 2d_x)
\]

where \( v_{cx} \) and \( S_x \) are the converter voltage and average switching function, respectively. Combining (2) and (3) yields

\[
\frac{V_m}{V_{dc}} = \frac{R_i i_{sx} + K v_{sx}}{v_{sx}} \Rightarrow i_{sx} = \left( \frac{V_m}{V_{dc}} - K \right) \frac{v_{sx}}{R_i} = \frac{v_{sx}}{R_i}
\]

where \( R_i \) represents the equivalent per phase resistance of the PWM rectifier when viewed from the ac input side (see Fig. 1). Equation (4) clearly shows that the currents drawn from the ac source are proportional to the supply voltages in the steady state. In other words, unbalance and distortion in the supply voltages will give rise to corresponding unbalanced and harmonic distortions in the input currents. This is consistent with the conclusion deduced in [3].

Moving on to analyze the harmonic effects on the dc output side, and by assuming that the converter is lossless, the corresponding power balance equation can then be written as,

\[
v_{cx} i_{x} + v_{bx} i_{bx} + v_{cx} i_{xc} = V_{dc} i_{dc}
\]
Substituting (3) to (5) subsequently gives rise to the following expression for the dc side current.

\[ i_{dc} = S_d i_d + S_b i_{sb} + S_i i_{sc} \]  

(6)

Next, by applying Clarke transformation to (6), (7) is yielded (notice that there is no zero sequence component exist in three-phase three-wire power network).

\[ i_{dc} = S_d i_d + S_b i_{sb} \]  

(7)

To find a closed form expression for \( i_{dc} \), the computation in (7) is better managed by expressing the \( abf \) variables in their complex forms, before deriving them from their individual harmonic components, orientated in their respective synchronous rotating \( dq \) frames. Upon performing that, the current variable in the \( abf \) coordinate can be written as,

\[ i_{\alpha} + j i_{\beta} = \sum_{n=-\infty}^{\infty} e^{j n \omega t} (i_{d}^{n} + j i_{q}^{n}) \]

\[ = \sum_{n=-\infty}^{\infty} (\cos(n \omega t) + j \sin(n \omega t)) (i_{d}^{n} + j i_{q}^{n}) \]

\[ = \sum_{n=-\infty}^{\infty} [(i_{d}^{n} \cos n \omega t - i_{q}^{n} \sin n \omega t) + j(i_{d}^{n} \sin n \omega t + i_{q}^{n} \cos n \omega t)] \]

(8)

where \( n \) is the order of harmonics in the line currents, and the sign of \( n \) determines whether it is positive- or negative-sequence. In the same manner, the average switching functions can be found as,

\[ S_{\alpha} + j S_{\beta} = \sum_{h=-\infty}^{\infty} e^{j h \omega t} (S_{d}^{h} + j S_{q}^{h}) \]

\[ = \sum_{h=-\infty}^{\infty} [(S_{d}^{h} \cos h \omega t - S_{q}^{h} \sin h \omega t) + j(S_{d}^{h} \sin h \omega t + S_{q}^{h} \cos h \omega t)] \]

(9)

where \( h \) is the order of harmonics in the average switching functions. It should be noted that \( h \) must be distinguished from \( n \) so as to generalize the interactions between all the ac components of different frequencies. Upon reaching (8) and (9), the relevant expressions governing \( i_{\alpha}, i_{\beta}, S_{\alpha}, \) and \( S_{\beta} \) can then be derived from the real and imaginary parts of the equations, before using them to calculate \( i_{dc} \) as follows.

\[ i_{dc} = \sum_{n, h=-\infty}^{\infty} [(S_{d}^{h} \cos h \omega t - S_{q}^{h} \sin h \omega t)(i_{d}^{n} \cos n \omega t - i_{q}^{n} \sin n \omega t)] \]

\[ = \sum_{n, h=-\infty}^{\infty} [(S_{d}^{h} i_{d}^{n} + S_{q}^{h} i_{q}^{n})(\sin h \omega t \sin n \omega t + \cos h \omega t \cos n \omega t)] \]

\[ + \sum_{h=-\infty}^{\infty} [(S_{d}^{h} i_{d}^{n} + S_{q}^{h} i_{q}^{n})(\cos h \omega t \sin n \omega t - \sin h \omega t \cos n \omega t)] \]

\[ + \sum_{n, h=-\infty}^{\infty} [(S_{d}^{h} i_{d}^{n} + S_{q}^{h} i_{q}^{n})(\cos h \omega t \sin n \omega t - \sin h \omega t \cos n \omega t)] \]

(10)

In OCC, the average switching function \( S \) has similar spectral content as the relevant line current (see eq. (1)), implying that (switching harmonics are not considered here since they can be filtered easily),

\[ \sum_{n, h=-\infty}^{\infty} (-\cos h \alpha \sin n \alpha + \sin h \alpha \cos n \alpha) = 0 \]  

(11)

By further manipulating the remaining trigonometric functions in (10), a simplified form for \( i_{dc} \) can be derived as,

\[ i_{dc} = \sum_{n, h=-\infty}^{\infty} (S_{d}^{h} i_{d}^{n} + S_{q}^{h} i_{q}^{n})(\sin h \alpha \sin n \alpha + \cos h \alpha \cos n \alpha) \]

\[ = \sum_{n, h=-\infty}^{\infty} (S_{d}^{h} i_{d}^{n} + S_{q}^{h} i_{q}^{n})[\cos(h-n)\alpha] \]

(12)

Since the amplitudes of harmonics are relatively small, it is reasonable to neglect the interactions between each harmonic. In this case, only the fundamental and coupling terms between the fundamental and harmonics are worthy of concern, and (12) can be further simplified as,

\[ i_{dc} = (S_{d}^{+1} i_{d}^{+1} + S_{q}^{+1} i_{q}^{+1}) \]

\[ + \sum_{n=-\infty}^{\infty} (S_{d}^{+1} i_{d}^{n} + S_{q}^{+1} i_{q}^{n})[\cos(1-n)\alpha] \]

\[ + \sum_{h=-\infty}^{\infty} (S_{d}^{-1} i_{d}^{h} + S_{q}^{-1} i_{q}^{h})[\cos(h-1)\alpha] \]

(13)

Obviously, the last two terms on the right hand side (RHS) of (13) can lead to voltage ripples on the dc output side of the rectifier, and it is easy to prove that they are related by the following equality.

\[ \sum_{n=-\infty}^{\infty} (S_{d}^{+1} i_{d}^{n} + S_{q}^{+1} i_{q}^{n})[\cos(1-n)\alpha] \]

\[ = \sum_{h=-\infty}^{\infty} (S_{d}^{-1} i_{d}^{h} + S_{q}^{-1} i_{q}^{h})[\cos(h-1)\alpha] \]

(14)

In [3], it is broadly pointed out that the \( n^{th} \) harmonic component found in the supply voltages can create harmonics of orders \( n-1, n+1 \) and \( 2n \) in the dc side current. Refining that conclusion, (13), being the closed form expression firstly derived for \( i_{dc} \) in this paper, conveys a more focused viewpoint, stating that only the \( |n-1| \) harmonic is the dominant component. For instance, in the power distribution network where some typical harmonics such as \( 5^{th} \) and \( 7^{th} \), \( n = h = -5,7 \) almost always exist, the dc side current will likely contain the \( 6^{th} \) harmonic component, and if the system is also unbalanced \( (n = h = -1) \), \( 2^{nd} \) harmonic is likely to be detected too. This point is later demonstrated by the experimental results presented in Section 4. In fact, the even harmonic components detected on the dc side may in turn affect the ac input currents, which further degrades the performance of PWM rectifier.
III. OPERATING PRINCIPLES OF THE PROPOSED OCC

The objectives defined for the proposed OCC are to provide a balanced set of nearly sinusoidal current waveforms under non-ideal supply conditions, and to minimize the dc side voltage ripples. For meeting these two objectives, the line currents measured for the inner current loop must be broken down into their fundamental and harmonic components, whose pictorial representation is drawn in Fig. 3. Mathematically, currents measured for the inner current loop must be broken down into their fundamental and harmonic components, whose harmonics coming from the source voltages, and whose source voltages, which yields, a balanced set of nearly sinusoidal current waveforms under respective harmonic source voltages, and then using the following first order high-pass filter (HPF) to block the dc component, which corresponds to the positive fundamental component in the abc frame.

Equation (17) shows that the implementation of the proposed OCC requires the extraction of harmonic components from the supply voltages. A straightforward method for realizing this step is to transform the three-phase voltages from abc frame to the fundamental positive synchronous dq frame, and then using the following first order high-pass filter (HPF) to block the dc component, which corresponds to the positive fundamental component in the abc frame.

$$H_{dq}(s) = \frac{s}{s + \omega_c}$$

where \(\omega_c\) is the cutoff frequency, which should be chosen small, around 10 rad/s, so as to avoid interference with the low order harmonics. Although the dq transformation method works fine, the extent of computational complexity involved compromises on the OCC simplicity, which has traditionally been stated as an advantage for the OCC. Therefore, instead of shifting the voltage variables, the equivalent approach taken in [10] is recommended, whereby the HPF is inversely shifted from the synchronous dq frame to the stationary \(αβ\) frame. Mathematically, the inverse shifting process can be initiated by applying (19).

$$H_{αβ}(s) = \begin{bmatrix}
H_{dq}(s + j\omega_b) & jH_{dq}(s + j\omega_b) \\
+H_{dq}(s - j\omega_b) & -jH_{dq}(s - j\omega_b) \\
+H_{dq}(s + j\omega_b) & H_{dq}(s + j\omega_b)
\end{bmatrix}$$

where \(\omega_b\) represents the fundamental frequency. Applying the inverse Clarke transformation and by substituting (18) into (19), the equivalent stationary frame HPF is derived as (20) with \(\alpha^2 \ll \alpha^2\).

$$H_{αβ}(s) = \begin{bmatrix}
\frac{s^2 + \omega_s^2 + \frac{\omega_s \omega_o}{\sqrt{3}} + \omega_b^2}{s^2 + 2\omega_s \omega_s^2 + \omega_b^2} & \frac{2\omega_s \omega_o}{\sqrt{3}} \\
\frac{2\omega_s \omega_o}{\sqrt{3}} & s^2 + 2\omega_s \omega_s^2 + \omega_b^2
\end{bmatrix}$$
Because of the equivalence in theories, (20) obviously can be used for extracting the needed harmonic and negative sequence signals from the supply voltages, like the dq transformation technique. However, (20) does have an advantage in the sense that it can conveniently be implemented either by using a universal filtering integrated chip [11] or as a modular subroutine for digital implementation [10]. The complete control diagram of the proposed OCC is then shown in Fig. 4, where further simplification can be introduced if the supply-side inductance \( L_c \) and filter capacitance \( C \) were set at 420 \( \mu \)F, deemed as an appropriately small value for the dc side after incorporating the proposed OCC control. Clearly illustrating any improvement that can be achieved on the ac input line voltage from a 1.2 kW boost type three-phase PWM rectifier was built in the laboratory. The existing OCC scheme used earlier is now replaced by the mathematical analysis presented in that section.

The second mathematical term is absent in (13) for calculating the third harmonic component observed here is not zero, since it is caused by the second harmonic sequence by nature, since it is caused by the second harmonic and to be more precise, the dc-link capacitive voltage is noted to vary with 2.87 V of second harmonic and 0.43 V of fifth harmonic, caused solely by the flow of line harmonic currents as explained in an earlier section. Indeed, these observations match well with the conceptual findings discussed in Section 2, hence raising the level of confidence for the experimental testing were performed with the programmable input supply set to include 10% of fundamental negative-sequence component (180° phase-shifted), 10% of fifth harmonic (in phase) and 5% of seventh harmonic (in phase). The resulting three-phase voltage waveforms are shown in Fig. 5, where the peak amplitude of phase \( a \) is observed to be 21.4 V lower than those of phases \( b \) and \( c \). Using the existing OCC scheme, the three-phase currents drawn from the ac source are severely distorted and unbalanced, as reflected by the waveforms captured in Fig. 6 (a). To reinforce that observation, Fig. 6 (b) shows the harmonic spectrum of the phase \( a \) current, where third, fifth and seventh harmonics are noted to be prominent, and sum to give a total harmonic distortion (THD) of 13.10%. Notice that the third harmonic component observed here is not zero-sequence by nature, since it is caused by the second harmonic component measured at the rectifier dc link. Other than second harmonic and to be more precise, the dc-link capacitive voltage is noted to vary with 2.87 V of second harmonic and 0.43 V of sixth harmonic, caused solely by the flow of line harmonic currents as explained in an earlier section. Indeed, these observations match well with the conceptual findings discussed in Section 2, hence raising the level of confidence for the mathematical analysis presented in that section.

The existing OCC scheme used earlier is now replaced by the modified OCC scheme proposed in this paper, and effected by using (17). As expected, the input current waveforms shown in Fig. 7 (a) are greatly improved with nearly sinusoidal and balanced wave shape observed throughout the captured time span. This improvement is equally observed in Fig. 7 (b), where the plotted spectrum clearly shows the reduction of third, fifth and seventh harmonics in the phase \( a \) current. The measured THD is also lower at 3.46%, which comparatively is a sizable improvement, judging from the severely distorted and unbalanced source voltages that the experiments have started off with. Adding on, the dc-link voltage observed in Fig. 7 (a) is much smoother than that captured in Fig. 6 (a). Explanation for that can be found in an earlier section, where it is specifically stated that the dc-link voltage would be smoother if the second mathematical term is absent in (13) for calculating \( i_{dc} \). In spectral context, the observed smoothness in dc-link
voltage from 0.43 V to nearly zero. Harmonic voltage from 2.87 V to 1.06 V, and its sixth harmonic voltage is mainly attributed to the reduction of its second harmonic voltage from 2.87 V to 1.06 V, and its sixth harmonic voltage from 0.43 V to nearly zero.

V. CONCLUSION

In this paper, the performance of existing OCC-based PWM rectifier under unbalanced and distorted supply conditions is studied. Specific finding discovered is that the input three-phase currents and the output dc voltage will contain harmonics under such operating conditions when no modification is made to the existing OCC-based control scheme. Dominant harmonics likely to be detected across the dc-link is computed mathematically, before a modified OCC control equation is proposed for guaranteeing balanced and sinusoidal three-phase input currents, and minimized dc-link voltage ripple. Relevant concepts on reference signal reconstruction in the OCC control loop, stationary-frame harmonic extraction, and experimental testing are all presented for validating the feasibility and practicality of the proposed OCC control scheme.

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