Direct Instantaneous Torque Control of a Four-Phase Switched Reluctance Motor

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Abstract — The paper presents an investigation on the direct instantaneous torque control (DITC) of a 1-hp four-phase 8/6 switched reluctance motor (SRM). In the proposed technique, the instantaneous phase torque was estimated in real-time using a pre-stored look-up table of measured static torque, \( T_i(\lambda) \), where phase current and phase flux linkage were input parameters. Therefore, a high-resolution encoder can be avoided. The ripple-minimizing reference phase torques were generated from the torque distribution and commutation sequence controllers that use the knowledge of the reference total torque, rotor position information obtained from an inexpensive low-resolution encoder and the estimated phase torques. The fully digitalized closed-loop control consisted of effective and simplified PI controllers of phase torques, which straightforwardly gave the suitable PWM duty cycles to the IGBT-based asymmetric half-bridge converter. Obviously, the instantaneous torque of the SRM was directly controlled by continuously variable phase voltages without including any inner current control loop. Results obtained from computer simulations of the developed MATLAB/Simulink model and the DSP-based real-time control experiments have confirmed the validity and acceptable performance of the proposed scheme. Good step torque responses and good tracking abilities to various torque reference waveforms have been presented.

Index Terms—reluctance machines, reluctance motor drives, reluctance motors, torque control

I. INTRODUCTION

Switched reluctance motors (SRM) have many advantageous characteristics comparing to those of the conventional AC and DC machines. The mechanical simplicity in construction of the SRM can be seen through their purely laminated-steel structure without permanent magnets, rotor windings and squirrel-cage bars. Thus, SR machines offer high reliability and robustness in operation. Due to their ruggedness, the SR motors are inherently suitable for high-speed drives and applications in high-temperature and hazardous environments. In addition, the SRM are efficient and suitable for some applications which required high torque and high dynamics. Consequently, the switched reluctance motors have recently gained a considerable attention from industries and researchers in the specific areas of high-performance and adjustable speed drives.

Nevertheless, SRM are very nonlinear in nature due to their operations in high-saturation conditions. The highly non-uniform reluctance torque is produced from magnetic saliency between stator poles and rotor poles. Phase flux linkages and instantaneous phase torque are nonlinear functions of phase currents and rotor positions. Therefore, without proper control, the inherent torque ripples, vibrations and acoustic noise can become major problems of the SRM drives. For a century, such drawbacks have prevented the SR motors from being widely used in applications of high-quality and variable speed drives. With the view to achieve high-performance servo drives for SRM, several instantaneous torque control techniques including torque ripple minimization features have been successively proposed in the last three decades.

In the conventional scheme called indirect instantaneous torque control (IITC), SRM torque is regulated by controlling the instantaneous phase currents in a cascaded fashion. The reference torque is converted to equivalent reference phase currents so that they can be tracked in the inner current control loops. By using the phase current profiling technique, optimal phase torques corresponding to torque sharing functions can be generated and, ultimately, torque ripples can be minimized. Various methods of torque ripple-minimization using IITC were extensively surveyed in Husain [1]. However, torque-to-current conversion in SRM is complex and becomes non-trivial due to their nonlinear relationship. Analytical expression of such conversion is complicated and leads to intensive on-line computation. On the other hand, the current profiles can be pre-calculated and pre-stored in the controller memory. Yet, this method requires large amount of on-line memory space.

Concepts of direct instantaneous torque control (DITC) for SRM have been developed to overcome the mentioned drawbacks of the IITC. The main DITC characteristics are: the instantaneous torque (which can be estimated from motor terminal quantities) is considered directly as a control variable, torque-to-current conversion and closed-loop control of phase currents are no longer required. In other words, DITC does not use current or torque profiles. Therefore, DITC is expected to counteract the torque error instantaneously with fast dynamic response and effectively minimize the inherent torque ripple. A DITC for SRM using the concept of a short flux pattern that links two separate poles of the stator was originally reported in Jinupun and Luk [2]. In order to achieve this flux pattern, a new type of winding configuration was proposed in this scheme. The new winding technique limits the length of two individual short...
II. INSTANTANEOUS TORQUE ESTIMATION

It is very difficult to find an accurate analytical representation for the instantaneous torque that is simple enough for real-time SRM model computation. Therefore, the experimentally measured static torque characteristics are used to develop an on-line SRM model in the look-up table form. The difference between the dynamic torque and the static torque is caused by the eddy current effects, magnetic hysteresis effects and the inter-phase coupling effects. Although, the method does not incorporate any dynamic changes in the machine parameters, the model can be considered reasonably accurate if the iron losses and mutual coupling effects are small enough, and can be neglected. This assumption is justified because the SRM has laminated-steel rotor and stator with a large ratio of yoke-to-pole width.

Instantaneous torque can be estimated from a look-up table of measured phase torque, where phase current and phase flux linkage are input variables, $T(i,\lambda)$, as presented in [9]. The flux linkage is estimated from measured phase voltage and phase current. The look-up table based modeling has significantly improved the real-time computation speed. The
$T(i,\lambda)$ model served as a torque observer of the SRM using only terminal measurements of voltage and current. Unlike the instantaneous torque estimation that uses a look-up table of measured phase torque, where phase current and rotor position are input variables, $T(i,\theta)$, the use of $T(i,\lambda)$ does not require a high-resolution position sensor.

### A. Instantaneous torque look-up table $T(i,\lambda)$

By combining the measured torque characteristics $T(i,\theta)$ and the measured flux linkage characteristics $\lambda(i,\theta)$, the need for the accurate position signal can be avoided. This is due to the fact that the rotor position in the torque characteristics $T(i,\theta)$ has been substituted with the corresponding flux linkage. This has resulted in the measured static torque characteristics $T(i,\lambda)$. In this case, the measured phase current and the estimated phase flux linkage are used as the row and the column pointers of the torque matrix, as shown in Fig. 7.1. As a result, by giving the phase current, $i_j$ and flux linkage, $\lambda_j$ as input parameters to the look-up table, the estimated phase torque, $T_j$ can be obtained using interpolation algorithm. By superposition principle, the total estimated torque of the SRM can be found by summing up the four individual estimated phase torques together. The negative torque can be predicted using the same look-up table where the torque characteristics are mirrored around the current-flux linkage plane. Acceptable performance of the developed instantaneous torque estimator has been confirmed both in computer simulations and experimental investigation.

![Fig. 1. Instantaneous phase torque estimation using the look-up table $T(i,\lambda)$](image)

### B. Flux linkage estimation

The proposed torque estimation technique requires flux linkage as an input variable. Therefore, the accuracy of the estimated torque is closely related to the accuracy of the estimated flux linkage. Basically, the flux linkage $\lambda_j$ of a certain phase can be estimated from the terminal quantities of that particular phase, i.e. the phase voltage $v_j$, the phase current $i_j$ and the phase winding resistance $R_j$. The phase flux linkage can be estimated by using the integral form of Faraday’s law, as presented in (1).

$$\lambda_j = \int (v_j - R_j \cdot i_j) \cdot dt . \quad (1)$$

Fig. 2 shows waveforms of measured phase currents, estimated phase flux linkages and the corresponding estimated phase torques of the four-phase SRM. Notice that at the end of each excitation stroke, phase currents and flux linkages reset to zero. The positive phase torques are in ABCD sequence and the total torque becomes +1.0 N·m.

![Fig. 2. Waveforms of instantaneous phase torque estimation using $T(i,\lambda)$](image)

### III. TORQUE DISTRIBUTION AND COMMUTATION CONTROL

The aim of the torque ripple minimization scheme is to carefully coordinate the individual phase torques during overlapped commutation in such a way that the desired total torque is accurately produced. Therefore in this section, the proposed technique stressing on simplified phase torque distribution control and commutation angle control. The algorithm does not incorporate any torque sharing functions (TSF) in order to avoid control implementation with intensive computation.

#### A. Torque distribution control

Although overlapped conduction in the SRM can help improve the quality of the total torque, plain commutation cannot significantly decrease the inherent torque ripples. In order to effectively achieve the reduced torque ripples, the controlled phase torques during commutation in the overlap zone should be adaptively distributed. The incoming phase would contribute the increasing phase torque while the outgoing phase would contribute the decreasing phase torque. For generalization, consider an actively controlled phase $j$. 

\[ T(i,\lambda) \]
Firstly, the neighboring phase torques \((T_{j-1} \text{ and } T_{j+1})\) are estimated using the look-up table \(T(i, \lambda)\) explained earlier. Then, the desired torque of that particular phase \(T_j^*\) is obtained by subtracting those estimated torques from the total torque reference \(T^*\). The reference phase torque can be expressed in (2).

\[
T_j^* = T^* - T_{j-1} - T_{j+1}
\]  

(2)

The proposed torque distribution control incorporating the on-line torque observer is self-adaptive and applicable for the whole cycle of each phase: from the beginning to the end of excitation stroke. Torque contribution sequence of phase \(j\) starts from: \(T_{j-1} \rightarrow T_j \rightarrow T_{j+1}\). The algorithm is universal and valid for all situations: when the commutating phases are in their active periods or even when the outgoing phase is switched off. The summation of the overlapped phase torque must equal to the total torque reference. Therefore, the generalized reference phase torques of the 4-phase SRM can be found from the following equations:

\[
\begin{align*}
T_A' &= T^* - T_D - T_B \\
T_B' &= T^* - T_A - T_C \\
T_C' &= T^* - T_B - T_D \\
T_D' &= T^* - T_C - T_A
\end{align*}
\]  

(3)

However, in order to produce a continuous unidirectional torque, at most, only two adjacent phases of the SRM are allowed to conduct. Consequently, the reference phase torques corresponding to the overlap zones can be summarized in Table I.

<table>
<thead>
<tr>
<th>Overlap zone</th>
<th>Control aim</th>
<th>Reference phase torque</th>
</tr>
</thead>
<tbody>
<tr>
<td>Phase A&amp;B</td>
<td>(T_A + T_B = T^*)</td>
<td>(T_A = T^* - T_B)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(T_B = T^* - T_A)</td>
</tr>
<tr>
<td>Phase B&amp;C</td>
<td>(T_B + T_C = T^*)</td>
<td>(T_B = T^* - T_C)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(T_C = T^* - T_B)</td>
</tr>
<tr>
<td>Phase C&amp;D</td>
<td>(T_C + T_D = T^*)</td>
<td>(T_C = T^* - T_D)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(T_D = T^* - T_C)</td>
</tr>
<tr>
<td>Phase D&amp;A</td>
<td>(T_D + T_A = T^*)</td>
<td>(T_D = T^* - T_A)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(T_A = T^* - T_D)</td>
</tr>
</tbody>
</table>

As an example, Fig. 3 shows an application of the simplified torque distribution control of the DITC in real-time implementation. The reference phase torques during commutation intervals are generated in such a way that the total instantaneous torque can become as close as possible to the demanded torque of \(-1.0\) N·m.

**Fig. 3** Reference phase torques generated by the torque distribution control

**B. Commutation control**

The commutation logic of the DITC is identical to that of the IITC. Yet, for the DITC, the commutation control can be implemented by using a much more simplified encoder. A low-resolution position sensor (such as an opto-interrupter with slotted disk) is adequate for generating the commutation signals. This is due to the fact that the instantaneous torque estimation of the DITC is independent from the knowledge of the rotor position, as explained in the previous section. The expensive high-precision encoder is no longer necessary. For a positive torque demand, the ABCD excitation sequence is generated, as presented in Fig. 4. The enable signal \((S_j=1)\) is in synchronism with the positive torque-producing zone.

**Fig. 4** Commutation signals for positive torque production.

Consequently, the phase torque references of the SRM are defined over the commutation control logic. When the commutation signals are enabled \((S_j=1)\) the reference phase torques are assigned according to the torque distribution rule. Otherwise, when the commutation signals are disabled \((S_j=0)\) the reference phase torques will be reset to zero. These can be expressed in the following equation.

\[
T_j^* = \begin{cases} 
T^* - T_{j-1} - T_{j+1} & \text{when } S_j = 1 \\
0 & \text{when } S_j = 0 
\end{cases}
\]  

(4)
IV. PROPOSED DIRECT INSTANTANEOUS TORQUE CONTROL

The control algorithm of the direct instantaneous torque control of the SRM is clearly explained in Fig. 5. The closed-loop controllers of phase currents and the high-resolution position encoder are excluded from the proposed DITC. Firstly, flux linkage of each phase is estimated from the corresponding measured phase voltage and current. Next, the measured phase current and the estimated phase flux linkage are used to estimate phase torque from the look-up table $T(i, \lambda)$, as previously explained. The low-resolution encoder gives a proper sequence of commutation signals which corresponding to the selected turn on angle, $\theta_{on}$ and turn off angle, $\theta_{off}$. The magnitude of the reference phase torque can be found using the simplified torque distribution control mentioned earlier. The input reference phase torques are compared with the feedback estimated phase torques. The DITC torque regulator directly translates such torque comparison into the reference phase voltages (or PWM duty cycles). Finally, the appropriate gate drive signals are generated for the 4-phase asymmetric half-bridge converter.

Justification of using the applied phase voltage to directly control the instantaneous torque of the switched reluctance machine is presented in the following section.

A. Linearization of the instantaneous torque in DITC

Normally, the nonlinear instantaneous torque of SR machines can be found from the co-energy principle, as expressed in (5). For simplification, the equation becomes (6). Thus, the instantaneous torque of the saturated SRM can be found from the product of the flux linkage derivative (w.r.t. rotor position) and the phase current, as shown in (7).

$$T = \frac{\partial W'(i, \theta)}{\partial \theta} = \frac{\partial}{\partial \theta} \int_{0}^{i} \lambda(i, \theta) di$$  \hspace{1cm} (5)

$$T \approx \frac{\partial \lambda(i, \theta)}{\partial \theta} \int_{0}^{i} di ,$$ \hspace{1cm} (6)

$$T \approx \frac{\partial \lambda(i, \theta)}{\partial \theta} \cdot i$$ \hspace{1cm} (7)

Consider the phase voltage equation of the SRM:

$$v = R \cdot i + \frac{d \lambda(i, \theta)}{dt}$$

$$= R \cdot i + \frac{\partial \lambda}{\partial i} \cdot \frac{di}{dt} + \frac{\partial \lambda}{\partial \theta} \cdot \frac{d\theta}{dt}$$

$$= R \cdot i + l(i, \theta) \cdot \frac{di}{dt} + \frac{\partial \lambda}{\partial \theta} \cdot \omega$$ \hspace{1cm} (8)

This incremental inductance $l(i, \theta)$ has significantly large value such that phase current can be assumed unchanged in one sampling period. Once the resistive drop is negligible and the current is considered constant, the phase voltage equation can be approximated as (9)

$$v \approx \frac{\partial \lambda}{\partial \theta} \cdot \omega$$ \hspace{1cm} (9)

Consequently, the instantaneous phase torque equation of the SRM can be simplified to:

$$T \approx \frac{i}{\omega} \cdot v$$ \hspace{1cm} (10)

The rotor speed and the phase current are assumed constant during the control cycle. As a result, the torque expression has been linearized and the phase voltage $v$ becomes an effective control variable for the DITC.

B. Instantaneous torque controller

The instantaneous torque in the DITC is directly regulated via the voltage PWM control, as shown in Fig. 5. In the proposed scheme, the PI controller with gain scheduler has been chosen as the closed-loop controller of the instantaneous phase torque. The torque error between the reference phase torque $T_j^*$ and the feedback torque $T_j$ (from the instantaneous torque estimator $T(i, \lambda)$) is sent to the PI regulator to create the reference phase voltage $u_j^*$, as expressed in (11).

$$u_j^* = K_{PT} \cdot (T_j^* - T_j) + K_{IT} \cdot \int (T_j^* - T_j) \cdot dt ,$$ \hspace{1cm} (11)

where $K_{PT}$ and $K_{IT}$ are the proportional gain and the integral gain of the instantaneous torque regulator, respectively. Therefore, based on the proposed direct instantaneous torque control, the reference phase voltages of the 4-phase SRM can be found from the following expressions:

$$u_A^* = K_{PT} \cdot (T_A^* - T_A) + K_{IT} \cdot \int (T_A^* - T_A) \cdot dt$$

$$u_B^* = K_{PT} \cdot (T_B^* - T_B) + K_{IT} \cdot \int (T_B^* - T_B) \cdot dt$$

$$u_C^* = K_{PT} \cdot (T_C^* - T_C) + K_{IT} \cdot \int (T_C^* - T_C) \cdot dt$$

$$u_D^* = K_{PT} \cdot (T_D^* - T_D) + K_{IT} \cdot \int (T_D^* - T_D) \cdot dt$$ \hspace{1cm} (12)

V. IMPLEMENTATION OF THE DITC FOR SRM DRIVES

A. Computer modeling and experimental setup

The proposed DITC was developed in MATLAB/Simulink environment for simulation purposes. The structure of the dynamic model is equivalent to the control diagram presented in Fig. 5. The DSP based real-time control was also implemented to verify the validity of the computer modeling, as well as the transient and steady-state performance of the DITC system. The experimental setup is illustrated in Fig. 6. The tested 1-hp 4-phase 8/6 SR motor (Magna Physics, USA)
was coupled through a torque sensor (Vibrac, USA) to a JR16M4CH ServoDisc dc machine (Kollmogen, USA), which functioned as a dc generator (supplying resistive load) and gave load torque. The phase windings of the SRM were connected to the 4-phase IGBT-based asymmetric half-bridge converter. Rotor position information from the encoder, together with phase currents and phase voltages signals from sensors were simultaneously read as control system inputs. Those signals were captured into the computer through the digital I/O and A/D channels of a DS1102 DSP controller card (dSPACE, Germany). Information from the absolute encoder was used to generate only simple commutation signals required by the DITC. Selected PWM switching frequency was 12.5 kHz. The sampling time of the real-time control program for DITC implementation was 250 µs (the sampling frequency achieved was 4 kHz).

**Fig. 5** Control diagram of the direct instantaneous torque control (DITC) for SRM drives

**Fig. 6** Experimental setup of the direct instantaneous torque control for the SRM drives

### B. Simulation and experimental results

Modeling and experimental results of the 4-phase 8/6 switched reluctance machine are shown in this section in order to examine the operational performance of the DITC. The validity and effectiveness of the proposed instantaneous torque control algorithm can be observed through steady state and dynamic operations.
1) Steady state performance

Fig. 7 shows the simulation results of the proposed DITC when the reference torque of $+1.0\ \text{N·m}$ was demanded ($\omega = 250\ \text{rpm}, \theta_{on} = -30^\circ, \theta_{off} = -7^\circ$). The DITC effectively reduced the torque ripples and precisely regulated the instantaneous torque. From an experiment, each instantaneous phase torque closely tracked the reference phase torque (Fig. 8). The total instantaneous torque is illustrated in Fig. 9. The experimental results show almost zero steady state errors. The maximum and the minimum torques recorded were 1.05 N·m and 0.95 N·m, respectively. In other words, torque ripple was approximately $\pm 5\%$ of the torque reference. Thus, the steady state performance of the DITC has achieved relatively close to the truly servo-grade quality (torque ripple $< \pm 2\%$ of the nominal torque).

2) Transient performance

Simulations and corresponding experiments have been conducted to confirm the transient performance of the DITC. Good step responses to square-wave reference torque (amplitude $= \pm 1.0\ \text{N·m}$, frequency $= 1.0\ \text{Hz}$) are clearly presented in Fig. 10 and Fig. 11. Furthermore, the tracking ability to the triangular-wave reference torque (amplitude $= \pm 1.0\ \text{N·m}$, frequency $= 1.0\ \text{Hz}$) are shown in Fig. 12 and Fig. 13. All of the simulation results agreed very well with the results from DSP based experiments. Obviously, waveforms of the reference instantaneous torque were accurately tracked. Summing up, good performance of the proposed direct instantaneous torque control scheme has been confirmed through simulations and experiments in both steady state and dynamic operations. The DITC has shown significant improvement in the performance of the instantaneous torque control for SRM without including any closed-loop current controller or high-resolution encoder.
VI. CONCLUSIONS

This paper presents a simplified direct instantaneous torque control for a 1-hp four-phase 8/6 SRM drive. The instantaneous torque is estimated using a torque-flux linkage-current lookup table obtained from static measurement without involving any complicated analytical expression of SRM torque. A high-resolution encoder is eliminated from the proposed DITC since the rotor position information is not used in torque estimation. Commutation signals obtained from a low-resolution encoder are adequate for the control scheme. The simplified torque distribution and commutation control is self-adaptive and can effectively minimize torque ripples. The method is generalized and can be extended to multiphase-torque-sharing operation. Gain-scheduled PI regulators are used in closed-loop control of instantaneous phase torques. The torque controllers directly calculate PWM duty cycles for the converter. The DITC is implemented without current and flux linkage are also excluded from the proposed system. The simulation and experimental results have been found with the good degree of accuracy in steady state operations, step responses and tracking abilities. Low torque ripples and fast torque dynamics have been observed. These have confirmed the acceptable performance of the proposed direct instantaneous torque control for switched reluctance drives.

REFERENCES