A CONTRACT FOR TWO MARKETPLACES WITH TWO DIFFERENT DEMAND UNCERTAINTIES

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ABSTRACT

This study considers a decentralized supply chain where a manufacturer supplies a newsvendor-type item to a retailer who sells the item in two different marketplaces. The manufacturer lowers the two wholesale prices and offers a buyback commitment to operate the chain as a centralized one. The buyback contract, however, is assumed to only accept one of the two marketplaces’ unsold products, which not only allows the manufacturer to collect the near marketplace’s returns if facing a same demand uncertainty but also determines which marketplace is advantageous to implement the return practice if facing two different demand uncertainties. The objective is to create a win-win situation via negotiating the two wholesale prices and the buyback price. The contributions are twofold. If the two marketplaces face a same demand uncertainty, our contract includes the existing complete-return contract as a special case. If the two marketplaces face two different demand uncertainties, our contract will determine which one of the two marketplaces should implement the return practice.

I. INTRODUCTION

It is well known that if a manufacturer and a retailer, two independent entities in a supply chain, are seeking to maximize profits, it will generate a so-called “double-marginalization” problem [1], leading the chain to a poor channel profit performances as a result of a lower optimal order quantity, compared to a coordinated supply chain. Therefore, contractual terms enhancing channel profit efficiency for a supply chain have become an imperative when dealing with inventory management. Two purposes of such contracts include supply chain coordination and Pareto-efficiency; a contract is called to coordinate a supply chain if it is set such that the chain’s expected profit is maximized, and a contract is said to be Pareto-efficient if the profit of each member in the chain is no worse off as the contract is in place by comparison with other default contracts [2]. Using a price-only contract as the benchmark, we will discuss a newsvendor supply chain with return policy consideration. The reason is given follows. In a traditional price-only contract, the manufacturer would not provide any incentive to retailers, and the wholesale price is their only decision. However, many researchers such as Larivieve & Porteus [3], Cachon [4] and Bernstein & Federgruen [5] have pointed out that a price-only contract cannot coordinate a supply chain. Contrary to that, a return-policy contract, mainly mitigating the risk of over-stocking due to demand uncertainty, is a commitment made by manufacturer, service provider or upstream distributor to accept their downstream member’s excess inventory at the end of selling season [6]. Pasternack [7], the first to analyze manufacturer-retailer channel coordination via a return policy for a seasonal item, has claimed that return policies could be used as an instrument for supply chain coordination. Thus, plenty of articles have engaged in the related return

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policies in a newsvendor supply chain setting. For example, Emmons and Gilbert [8] investigated the role of return policies in pricing and inventory decisions for catalogue goods. Lau et al. [9] dealt with the problem of demand uncertainty and return policy for a seasonal product. Tsay [10] studied the quantity flexibility contract in a newsvendor supply chain. Yao et al. [11] tackled demand uncertainty and manufacturer return policies for style-good retailing competition. Bose and Anand [2] contributed to a research on return policies with exogenous price. Yao et al. [12] analyzed the impact of price-sensitivity factors on return policy in coordinating supply chain. Chen [13] considered return policies with wholesale-price-discount contract in a newsvendor problem, and Devangan et al. [14] developed individually rational buyback contracts in supply chain coordination with an inventory level dependent demand in which they found that the existing buyback contracts in the literature do not guarantee the satisfaction of individual rationality constraints. Additionally, Wang et al. [15] dealt with inventory management and pricing decisions for a newsvendor supply chain considering demand leakage and a return-policy contract. Meanwhile, Zhen & Negenborn [16] proposed a negotiation model to represent a supplier and a buyer under uncertainty about the supplier’s cost and the buyer’s revenue when facing fixed and elastic demands. Also, Qi et al. [17] built game theoretic models to investigate a one-manufacturer-two-retailer supply chain in which the two retailers compete with each other by selling a newsvendor-type item in an uncertain demand market.

In general, a complete-return policy allowing a manufacturer to buy back all unsold products and a partial-return policy (also known as a quantity flexibility contract) only accepting returns no more than a certain percentage of the order quantity are two types of return policies comprehensively explored in the literature. These two contractual forms were later proven equivalent by Lariviere [18]. We consider a decentralized supply chain that the retailer sells an item in two different marketplaces with different demand uncertainties. In order to operate the chain as a centralized supply chain, the manufacturer lowers the two wholesale prices and offers a buyback contract to the retailer. The contract, however, only accepts one of the two marketplaces’ returns, and the retailer then takes all responsibilities for the other marketplace’s over-stock. The motivation of our contract is based on the following situations. First, it is commonly seen that a retailer could sell a single item in two different marketplaces where are far away from each other. Thus, if the manufacturer is allowed to only accept one of the two marketplaces’ unsold products, he could choose the near marketplace’s returns if facing a same demand uncertainty; and this will save both members a lot of return-collecting time and administration cost. Second, it is also commonly seen that each of the two marketplaces owns its demand uncertainty. Thus, if the manufacturer is allowed to only accept one of the two marketplaces’ unsold products, our contract will determine which marketplace is advantageous to implement the return practice if the two demand uncertainties are different. Third, if the manufacturer is allowed to only accept one of the two marketplaces’ unsold products, how the buyback price should be negotiated to offset the other marketplace’s unreturned inventory losses. Therefore, our return policy is quite distinct from the existing return-policy contracts. To the best of our knowledge, we are the first to respond to these situations.

During the course of this study, we will prove that our contract not only provides both manufacturer and retailer with a flexibility to determine which one of the two marketplaces is more advantageous to implement the return practice; our contract also coordinates the decentralized supply chain as well as creates a win-win (Pareto efficiency) situation to both members. More importantly, our conducted numerical examples will prove that our contract includes the extant complete-return contract as a special case if the two marketplaces face a same demand uncertainty.

The remainder of this study is organized as follows. Assumptions and notations are given in Section II where relevant models responding to our subject are proposed, along with theoretical analysis to draw significant conclusions. Solution method and numerical examples are demonstrated in Section III. Finally, a summary and contributions of this study and potential directions for further explorations are presented in Section IV to complete the study.

II. MODELS

The scenario of this study is described as follows. A manufacturer, who is a Stackelberg game leader, supplies a newsvendor-type item to a retailer who sells the item in two marketplaces. According to Chen [13], the following assumptions are made. For \( i = 1, 2, x_i = D_i + \varepsilon_i \) is the marketplace \( i \)'s stochastic demand, where \( D_i \) is the deterministic demand and \( \varepsilon_i \) is the demand uncertainty defined in the range of \( [A_i, B_i] \) with \( -A_i \leq D_i \) to ensure the non-negative demand \( x_i \). Define \( F_i(\cdot) \) as the probability
density function, $F_i(\cdot)$ as the cumulative distribution function that is increasing and invertible with $F_i'(\cdot)$ as its inverse function, and $\mu_i$ as the mean of $e_i$. Also, $p_i$ is the unit retail price, $s_i$ is the unit shortage cost for unsatisfied demand, zero salvage value is assumed for over-stock, and the unit production cost $c$ is incurred for the manufacturer.

1. Price-Only Contract

We start with a traditional price-only contract in which the retailer decides his order quantity $Q_i$, $i = 1, 2$, after the manufacturer’s announcement of wholesale price $w_i$ before the selling period, and then takes all risk of the two marketplaces’ unsold products at the end of the selling period. Then, according to Thowsen [19] and Petruzzi & Dada [20], we let $Q_i = D_i + z_i$ with $z_i$ as a safety stock level, the profit of marketplace $i$ is then calculated by

$$z_i^* = (p_i - w_i)(D_i + \mu_i) - w_i(z_i - \mu_i) - (p_i - w_i + s_i)(e_i - z_i)$$

where $(\cdot) = \max\{0, \cdot\}$. And its expected profit is obtained by

$$E[\pi_i^w] = (p_i - w_i)(D_i + \mu_i) - w_i\mu_i - (p_i - w_i + s_i)\Theta(z_i)$$

where $\Theta(z_i) = \int_{z_i}^{\infty} e_i f(e_i) \, de_i$ and $\mu_i = \int_{0}^{\infty} e_i f(e_i) \, de_i$ imply expected overstock and expected understock, respectively. Thus, the retailer’s overall expected profit, denoted by $E[\pi_i^w]$, in the price-only contract is $E[\pi_i^w] = E[\pi_1^w] = E[\pi_2^w]$ and is given by

$$E[\pi_i^w] = \sum_{i=1}^{2} (p_i - w_i)(D_i + \mu_i) - \sum_{i=1}^{2} w_i\mu_i - \sum_{i=1}^{2} (p_i - w_i + s_i)\Theta(z_i)$$

The first term in Equation (3) implies the retailer’s sales profit, and the second term and the third term imply overstock losses and shortage cost, respectively. The retailer’s objective is to find the optimal $z_i$, $i = 1, 2$, to maximize $E[\pi_i^w]$ as the wholesale prices $w_i$ are given. We thus take the first- and second-order derivatives w.r.t $z_i$ from Equation (3), yielding the following equations and thereby Proposition 1.

$$\frac{\partial E[\pi_i^w]}{\partial z_i} = (p_i - w_i + s_i) - (p_i + s_i)F(z_i)$$

$$\frac{\partial^2 E[\pi_i^w]}{\partial z_i^2} = -(p_i + s_i)f(z_i), \quad \frac{\partial^2 E[\pi_i^w]}{\partial z_i \partial \Theta(z_i)} = 0$$

for $i = 1, 2$

(4)

Proposition 1: $E[\pi_i^w]$ is jointly concave in $z_i$, $i = 1, 2$, and the optimal $z_i$ and the optimal $Q_i$, denoted by $z_i^*$ and $Q_i^*$, respectively, uniquely exist in the form of

$$z_i^* = F_i^{-1}\left(\frac{p_i - w_i + s_i}{p_i + s_i}\right)$$

$$Q_i^* = D_i + \frac{p_i - w_i + s_i}{p_i + s_i}$$

(5)

(6)

Proof: See Appendix for details.

As for the manufacturer’s expected profit, it is obtained by

$$E[\pi_i^w] = \sum_{i=1}^{2} (w_i - c)(D_i + z_i^*)$$

(7)

The manufacturer’s objective is to find the optimal $w_i$, $i = 1, 2$, that maximize $E[\pi_i^w]$. Before proceeding to the problem of optimization, we recall the hazard rate $r(\cdot) = \frac{f(\cdot)}{1 - F(\cdot)}$ introduced by Barlow & Proschan [21]. The hazard rate roughly represents a percentage decrease in the probability of stock-out from increasing stocking quantity by one unit, according to Lariviere & Porteus [3]. According to Bagnoli & Bergstrom [22], many distributions such as normal, uniform, logistic, chi-squared and exponential distributions are all in the class of non-decreasing hazard rate distribution $r(\cdot) + r^2(\cdot) > 0$. With the aid of the hazard rate, the following outcome can be concluded.

Proposition 2: For $i = 1, 2$, if the demand uncertainty $e_i$ is in the class of non-decreasing hazard rate distribution, then the manufacturer’s $E[\pi_i^w]$ is jointly concave in $w_i$, and the optimal $w_i$, denoted by $w_i^*$ uniquely exists in the form of

$$w_i^* = \frac{c}{1 - (D_i + z_i^*)r(z_i^*)}, \quad i = 1, 2$$

(8)

Proof: See Appendix for details.

Consequently, if we substitute this $w_i^*$ into Equations...
and (6), we obtain the retailer’s optimal \( z_i^* \) and the optimal \( Q_i^* \) in the price-only contract as follows

\[
 z_i^* = F_i^{-1}\left( \frac{p_i - w_i^* + s_i}{p_i + s_i} \right) \quad (9)
\]

\[
 Q_i^* = D_i + F_i^{-1}\left( \frac{p_i - w_i^* + s_i}{p_i + s_i} \right) \quad (10)
\]

2. Centralized Supply Chain

We now consider that the chain is operated as a centralized supply chain in which we could assume that the manufacturer himself can sell the item. Thus, the chain profit is obtained by \( E[\pi^*] = E[\pi_u^*] + E[\pi_o^*] \). First, \( z_i = \lambda_i(z_i) - \Theta_i(z_i) + \mu_i \) is easy to prove; thus, according to Equations (3) and (7), \( E[\pi^*] \) is then given by

\[
 E[\pi^*] = \sum_{i=1}^{2} (p_i - c)(D_i + \mu_i) - \sum_{i=1}^{2} c \lambda_i(z_i) - \sum_{i=1}^{2} (p_i - c + s_i)\Theta_i(z_i) \quad (11)
\]

Similar to \( E[\pi_u^*] \) in the price-only contract, the optimal \( z_i \) and optimal \( Q_i \) in the centralized supply chain, denoted by \( z_i^* \) and \( Q_i^* \), respectively, are obtained by

\[
 z_i^* = F_i^{-1}\left( \frac{p_i - c + s_i}{p_i + s_i} \right) \quad (12)
\]

\[
 Q_i^* = D_i + F_i^{-1}\left( \frac{p_i - c + s_i}{p_i + s_i} \right) \quad (13)
\]

By comparison, the phenomenon of double marginalization in the price-only contract \( z_i^* > z_i^* \) and \( Q_i > Q_i^* \) is obtained because \( w_i^* > c \) and the increasing \( F_i^{-1}(\cdot) \), according to Equations (9), (10), (12) and (13).

3. Our Return Contract

In our return contract, to entice the retailer’s order up to the level of \( Q_i^* \), \( i = 1, 2 \), as in the centralized supply chain, not only will the manufacturer offer cheaper wholesale prices \( w_i^* \) compared to \( w_i^* \), \( i = 1, 2 \) in the price-only contract, he also provides a buy-all-back commitment. The commitment is assumed to only accept the second marketplace’s unsold products at a unit buyback price \( b \). The retailer takes full responsibilities of the first marketplace’s over-stock. The objective of our contract is to coordinate the chain and create a win-win situation by means of negotiating the two wholesale prices \( w_i^* \), \( i = 1, 2 \), and the second marketplace’s buyback price \( b \).

Thus, according to our contract, the retailer’s expected profit, denoted by \( E[\pi_u^*(W^*, z^*)] \), is obtained by substituting \( w_i = w_i^* \) and \( z_i = z_i^* \) into Equation (3) and adding the second marketplace’s return value \( b\lambda_i(z_i^*) \) as follows

\[
 E[\pi_u^*(W^*, z^*)] = \sum_{i=1}^{2} (p_i - w_i^*)(D_i + \mu_i)
 \]

\[
 - \sum_{i=1}^{2} w_i\lambda_i(z_i^*) - \sum_{i=1}^{2} (p_i - w_i^* + s_i)\Theta_i(z_i^*)
 \]

\[
 + b\lambda_i(z_i^*) \quad (14)
\]

Similarly, the manufacturer’s expected profit, denoted by \( E[\pi_u^*(W^*, z^*)] \), is obtained by substituting \( w_i = w_i^* \) and \( z_i^* = z_i^* \) into Equation (7) and subtracting the second marketplace returns buyback cost \( b\lambda_i(z_i^*) \) as follows

\[
 E[\pi_u^*(W^*, z^*)] = \sum_{i=1}^{2} (w_i - c)(D_i + z_i^*) - b\lambda_i(z_i^*) \quad (15)
\]

where \( W^* = (w_1^*, w_2^*) \) and \( z^* = (z_1^*, z_2^*) \). First, Equations (14) and (15) suggest that the retailer’s profit \( E[\pi_u^*(W^*, z^*)] \) increases in \( b \) but decreases in \( w_i \), whereas the manufacturer’s profit \( E[\pi_u^*(W^*, z^*)] \) increases in \( w_i \) but decreases in \( b \). Moreover, Equations (14), (15) and (11) imply \( E[\pi_u^*(W^*, z^*)] = E[\pi_u^*(W^*, z^*)] + E[\pi_o^*(W^*, z^*)] \) where \( E[\pi_o^*(z^*)] \) denotes the optimal profit in the centralized supply chain, and this suggests that our contract coordinates the supply chain. Additionally, both members’ profits in our contract should be better than those in the price-only contract to create a win-win situation, which means that both \( E[\pi_u^*(W^*, z^*)] \geq E[\pi_u^*(W^*, z^*)] \) and \( E[\pi_u^*(W^*, z^*)] \geq E[\pi_u^*(W^*, z^*)] \) are needed, where \( E[\pi_u^*(W^*, z^*)] \) and \( E[\pi_u^*(W^*, z^*)] \) denote the optimal retailer’s and manufacturer’s expected profits in the price-only contract, respectively. Therefore, to assure the retailer’s better profit \( E[\pi_u^*(W^*, z^*)] \geq E[\pi_u^*(W^*, z^*)] \), we define \( U \) as the retailer’s revenue increase when the order increases from \( z^* \) to \( z^* \) by solving the following equation:

\[
 U = \sum_{i=1}^{2} (p_i - \mu_i) - \sum_{i=1}^{2} (p_i - s_i)\Theta_i(z_i^*)
 \]

\[
 + b\lambda_i(z_i^*) - E[\pi_u^*(W^*, z^*)] \quad (16)
\]

We note that \( U \) is a constant once \( b \) is given. For simplicity, let \( Q_i = D_i + z_i^* ; \) we then obtain \( Q_i = D_i + z_i^* = \)
Thus, if substitute this result into \( \sum_{i=1}^{2} w_i Q_i \leq U \), it then implies \( E[\pi_i' (W^*, z')] \geq E[\pi_i' (W^*, z^*)] \), according to Equations (14) and (16). This reveals that if the retailer’s purchasing cost \( \sum_{i=1}^{2} w_i Q_i \) is offset by the revenue increase \( U \), then our contract is profitable to the retailer. Consequently, the retailer should negotiate \( w_i' \), \( i = 1, 2 \) such that \( \sum_{i=1}^{2} w_i' Q_i \leq U \) to ensure better profit.

Likewise, to ensure the manufacturer’s better profit \( E[\pi_u' (W^*, z')] \geq E[\pi_u' (W^*, z^*)] \), we define \( L \) as the manufacturer’s cost increase when the order increases from \( z' \) to \( z \) by the following equation

\[
L = \sum_{i=1}^{2} c_i Q_i + b_i \Lambda_i(z') + E[\pi_u' (W^*, z')]
\]  

(17)

Note that \( L \) is a constant once \( b \) is given. Then, \( E[\pi_u' (W^*, z')] \geq E[\pi_u' (W^*, z^*)] \) will be obtained if \( \sum_{i=1}^{2} w_i' Q_i \geq L \) holds, according to Equation (15). This suggests our profitable contract to the manufacturer if revenue \( \sum_{i=1}^{2} w_i' Q_i \) outperforms the cost increase \( L \). Consequently, the manufacturer should negotiate \( w_i' \), \( i = 1, 2 \) such that \( \sum_{i=1}^{2} w_i' Q_i \geq L \) to ensure better profit.

Meanwhile, according to Equations (16) and (17), it is learned that both \( U \) and \( L \) are increasing in \( b \), and \( U > L \) because \( U - L = E[\pi' (z')] - E[\pi' (z^*)] > 0 \) is a constant and independent of \( b \). Consequently, the following observations about the two negotiated wholesale prices are claimed.

**Proposition 3:** Compared to the price-only contract, if \( w_i' \), \( i = 1, 2 \), are negotiated such that

i. \( \sum_{i=1}^{2} w_i' Q_i \leq U \), the retailer will receive more profit from the game;

ii. \( \sum_{i=1}^{2} w_i' Q_i \geq L \), the manufacturer will receive more profit from the game;

iii. \( L \leq \sum_{i=1}^{2} w_i' Q_i \leq U \), both members will receive more profits from the game, making our contract a win-win situation.

We have proven that our contract achieves supply chain coordination and is win-win if the two negotiated wholesale prices \( w_i' \), \( i = 1, 2 \) satisfy \( L \leq \sum_{i=1}^{2} w_i' Q_i \leq U \) with \( 0 \leq w_i' \leq w_i' \). We now will provide a buyback price that the retailer can offset his first marketplace’s unreturned inventory losses as follows. Inspired by \( \sum_{i=1}^{2} w_i' Q_i \leq U \) in Proposition 3, we define \( \bar{b} \) as the retailer’s minimal buyback price by solving the following equation

\[
\sum_{i=1}^{2} w_i' Q_i = \sum_{i=1}^{2} p_i (D_i + \mu_i) - \frac{2}{\Lambda_i(z')} + b_i \Lambda_i(z') - E[\pi_u' (W^*, z')]
\]  

(18)

Then, from Equation (3) and Equation (18), \( b \) is obtained by

\[
b = \frac{1}{\Lambda_i(z')} \left( \sum_{i=1}^{2} w_i' \Lambda_i(z') - \Lambda_i(z') \right)
+ \frac{1}{\Lambda_i(z')} \left( \sum_{i=1}^{2} (p_i - w_i') \left( \Theta_i(z) - \Theta_i(z') \right) \right)
\]

(19)

Clearly, if the buyback price \( b \geq b_r \), we obtain \( \sum_{i=1}^{2} w_i' Q_i \leq U \), and thus \( \sum_{i=1}^{2} w_i' Q_i \leq U \leq U \) holds. As a consequence, the retailer can negotiate a buyback price that is higher than \( b \), to offset his first marketplace’s unreturned inventory losses and reach a win-win situation, according to Proposition 3.

Likewise, we will provide a buyback price that the manufacturer can endure during the game as follows. Inspired by \( \sum_{i=1}^{2} w_i' Q_i \geq L \) in Proposition 3, we define \( \tilde{b} \) as the manufacturer’s maximal buyback price by solving the following equation.

\[
\sum_{i=1}^{2} w_i' Q_i = \sum_{i=1}^{2} c_i Q_i + \bar{b} \Lambda_i(z_i) + E[\pi_u' (W^*, z')]
\]  

(20)

From Equations (7) and (20), \( \tilde{b} \) is obtained by

\[
\tilde{b} = \frac{1}{\Lambda_i(z_i)} \sum_{i=1}^{2} (w_i' - c_i)(z_i' - z_i')
\]  

(21)

Clearly, \( b > \tilde{b} \) implies \( \sum_{i=1}^{2} w_i' Q_i < L \) and then \( \sum_{i=1}^{2} w_i' Q_i \leq U \leq \sum_{i=1}^{2} w_i' Q_i < L \), which suggests that our contract is no longer profitable to the manufacturer, according to Proposition 3. As a result, the manufacturer should negotiate a buyback price lower than \( \tilde{b} \) to ensure his better expected profit. Furthermore, \( \tilde{b} > b_r \) is obtained because \( \tilde{b} > b_r = \frac{E[\pi'(z')]}{\Lambda_i(z_i)} \geq 0 \), according to Equations (19) and (21). Altogether, we summarize the following remarks responding to the buyback price \( b \).

**Proposition 4:** In response to the buyback price, the following are observed.

i. Basically, both members can negotiate a buyback price
the negotiated wholesales contain the following two graphics to illustrate the range of considering the first marketplace’s unreturned products for $b < b_r$.

To explain Figs. 1 and 2, the lines $\overline{AB}: \sum_{i=1}^{2} w_i'Q'_i = U$ and $\overline{CD}: \sum_{i=1}^{2} w_i'Q'_i = L$ are clarified. We recall that $U - L = E[\pi'(z')] - E[\pi'(z^*)]$ remains unchanged despite the value of $b$. This graphically implies that as $b$ increases, the two parallel line segments $\overline{AB}$ and $\overline{CD}$ will simultaneously move toward the right side of wholesale-price-limited region $[0, w'_1] \times [0, w'_2]$ at the same speed (see Fig. 1), and then $\overline{AB}$ may lie outside the region if $b < b \leq \overline{b}$ (see Fig. 2); eventually, both $\overline{AB}$ and $\overline{CD}$ are all beyond the region if $b > \overline{b}$. Thus, Figs. 1 and 2 conclude the following results.

**Proposition 5:** If the set of wholesale prices $(w'_1, w'_2)$ is negotiated such that

i. It lies in the region I of Fig. 1 or of Fig. 2, our contract is win-win;

ii. It lies in the region II of Fig. 1 or of Fig. 2, our contract favors the retailer’s profit but damages the manufacturer’s profit;

iii. It lies in the region III of Fig. 1, our contract damages the retailer’s profit but favors the manufacturer’s profit.

Furthermore, both members’ profit functions $E[\pi'_i(W', z')]$ and $E[\pi'_m(W', z')]$ can be re-written as follows. According to Equations (14) and (15)

$$E[\pi'_i(W', z')] = \frac{\sum_{i=1}^{2} w_i'Q'_i + \sum_{i=1}^{2} P_i(Q'_i - \Lambda_i(z'_i))}{L} - \frac{\sum_{i=1}^{2} \sigma_i(z'_i) + b\Lambda_i(z'_i)}{L}$$

(22)

$$E[\pi'_m(W', z')] = \frac{\sum_{i=1}^{2} w_i'Q'_i - \sum_{i=1}^{2} cQ'_i - b\Lambda_i(z'_i)}{L}$$

(23)

Clearly, both profit functions are linear in $w'_i$, according to Equations (22) and (23), which are subject to the linear constraints as stated in Proposition 3. Accordingly, the optimal retailer’s profit $E[\pi'_i(W', z')]$ will occur at any point of $\overline{CD}$ in Fig. 1 as $b < b_r$, or in Fig. 2 as $b \leq b \leq \overline{b}$, whereas the optimal manufacturer’s profit $E[\pi'_m(W', z')]$ will occur at any point of $\overline{AB}$ in Fig. 1 as $b < b_r$ or at point $A(w'_1, w'_2)$ in Fig. 2 as $b \leq b \leq \overline{b}$. Additionally, the location of $(w'_1, w'_2)$ determines which member benefits more from the game. If the retailer has more negotiation power, he will negotiate $(w'_1, w'_2)$ close to $\overline{CD}$ in Figs. 1 and 2. Contrarily, if the manufacturer dominates the game, he will negotiate $(w'_1, w'_2)$ close to $\overline{AB}$ in Fig. 1 or to point
\( P(w_i', w_j') \) in Fig. 2.

### III. EXAMPLES

To demonstrate that our one-marketplace-return contract in two different marketplaces setting reaches the same goals as Chen’s [13] complete-return contract in a single marketplace setting, we deliberately adopt the assumed parameter values as follows. For \( i = 1, 2, D_i = 200, p_i = 18, s_i = 1, c = 4, \) the demand uncertainty \( \varepsilon_i \) is normally distributed in the range of \([-200, 200]\) with mean \( \mu_i = 0 \) and standard deviation \( \sigma_i = 50 \).

As a result, optimal values in the price-only contract are obtained as follows: \((w_i', w_j') = (16.34, 16.34), (\zeta_i^*, \zeta_j^*) = (-54, -54), (Q_i^*, Q_j^*) = (146, 146), E[\pi_i(W^*, z^*)] = 241.31, E[\pi_j(W^*, z^*)] = 3603.28\) and the chain profit \( E[\pi'(W', z')] = 3844.59 \). Optimal values in the centralized supply chain are obtained as follows: \((\zeta_i^*, \zeta_j^*) = (40.23, 40.23), (Q_i^*, Q_j^*) = (240.23, 240.23)\) and the chain profit \( E[\pi'(z')] = 5051.89 \).

We first compare our contract with Chen’s [13] complete-return contract in a single marketplace setting in which the assumed buyback price is \( b = 5 \) and \( b = 7 \), respectively. We thus need to assume our buyback price \( b = 10 \) and \( b = 14 \), respectively, to reflect our one-marketplace-return contract in our two marketplaces setting. Our two negotiated wholesale prices are also intentionally set by \( w_i' = w_j' = 14.5 \) in accordance with the assumed wholesale price \( w^* = 14.5 \). In this situation, results indicate our \( E[\pi_i'(W', z')] = 468.9, E[\pi_j'(W', z')] = 4582.99 \) as \( b = 10 \) and \( E[\pi_i'(W', z')] = 653.63, E[\pi_j'(W', z')] = 4398.26 \) as \( b = 14 \), all of which are double the amount in Chen’s [13] results. 

\( E[\pi_3] = 234.45, E[\pi_4] = 2291.45 \) as \( b = 5 \) and \( E[\pi_5] = 326.72, E[\pi_6] = 2199.07 \) as \( b = 7 \), respectively. These outcomes confirm that our one-marketplace-return contract reaches the same goals as Chen’s [13] complete-return contract does if the two marketplaces face a same market demand uncertainty.

We next conduct our examples in Tables 1 and 2 in which the negotiated buyback price \( b_i \leq b < \bar{b} \) is assumed to feature our one-marketplace-return contract. In Table 1, \( b_i = 24.21 \) and \( \bar{b} = 50.35 \) are obtained, according to Equations (19) and (21); the values of \( U \) and \( L \) in Equations (16) and (17) will be obtained according to the negotiated \( b \). Table 1 assumes that the two marketplaces face a same market demand uncertainty, and negotiates several negotiated \( (w_i', w_j') \) satisfying \( L \leq \sum_{i=1}^{2} w_i'Q_i' \leq U \) to inspect how both the negotiated wholesale prices and the buyback price will impact both members’ profits. Table 2 examines which marketplace should implement the return practice when the two marketplaces face different market demand uncertainties as \( b = 28 \).

Table 1 confirms that a higher \( b \) benefits the retailer’s \( E[\pi_i'(W', z')] \) but impairs the manufacturer’s \( E[\pi_m(W^*, z^*)] \), and a higher \( (w_i', w_j') \) profits the manufacturer’s \( E[\pi_m'(W', z')] \) but damages the retailer’s \( E[\pi_i'(W', z')] \). Taking \((w_i', w_j') = (15, 15)\) as an example, \( E[\pi_i'(W', z')] \) increases from 967.61 at \( b = 26 \) to 1059.98 at \( b = 28 \) and then 1152.35 at \( b = 30 \), but at the same time \( E[\pi_m'(W', z')] \) decreases from 4084.28 to 3991.91 and then 3899.55. Conversely, for a given \( b = 26 \), \( E[\pi_m'(W^*, z^*)] \) increases from 3603.82 at \((w_i', w_j') = (14, 14)\) to 4084.28 at \((w_i', w_j') = (16, 16)\), whereas \( E[\pi_i'(W', z')] \) drops dramatically from 1448.07

### Table 1 Profit performances for \( \varepsilon_i, \varepsilon_j \in [-200, 200] \) at \( b = 26, 28, 30 \)

<table>
<thead>
<tr>
<th>( (b, L, U) )</th>
<th>( (w_i', w_j') )</th>
<th>( E[\pi_i'(W', z')] )</th>
<th>( E[\pi_j'(W', z')] )</th>
</tr>
</thead>
<tbody>
<tr>
<td>((26,6725.9, 7933.1))</td>
<td>((14,14))</td>
<td>1448.07</td>
<td>3603.82</td>
</tr>
<tr>
<td>((15,15))</td>
<td>1459.16</td>
<td>3622.24</td>
<td></td>
</tr>
<tr>
<td>((16,16))</td>
<td>1509.16</td>
<td>3664.74</td>
<td></td>
</tr>
<tr>
<td>((28,6818.3, 8025.9))</td>
<td>((15,15))</td>
<td>1059.98</td>
<td>4084.28</td>
</tr>
<tr>
<td>((15,15))</td>
<td>1064.87</td>
<td>4091.93</td>
<td></td>
</tr>
<tr>
<td>((16,16))</td>
<td>1105.98</td>
<td>4098.14</td>
<td></td>
</tr>
<tr>
<td>((30,6910.6, 6117.9))</td>
<td>((15,15))</td>
<td>1152.35</td>
<td>3899.55</td>
</tr>
<tr>
<td>((15,15))</td>
<td>1157.35</td>
<td>3906.78</td>
<td></td>
</tr>
<tr>
<td>((16,16))</td>
<td>1192.35</td>
<td>3913.01</td>
<td></td>
</tr>
</tbody>
</table>

### Table 2 Profit performances with the different \( \varepsilon_i, \varepsilon_j \) at \( b = 28 \)

| \((w_i', w_j') \) | \( \varepsilon_i \in [-200, 200], \varepsilon_j \in [-100, 100] \) | \( \varepsilon_i \in [-100, 100], \varepsilon_j \in [-200, 200] \) |
|----------------|----------------|----------------|----------------|
| \((14,16)\) | 1061.40 | 1079.41 |
| \((15,15)\) | 1067.55 | 1076.35 |
| \((16,14)\) | 4051.86 | 4033.81 |
| \((14,16)\) | 1064.77 | 4036.90 |
| \((15,15)\) | 4045.88 | 4039.99 |
to 967.61 and finally 487.16.

Although our contract only accepts the second marketplace’s returns, Table 2 shows that which marketplace should implement the return practice from the retailer’s (manufacturer’s) standpoint if facing two different demand uncertainties. Thus, Table 2 compares two different wholesale prices that make our contract a win-win situation; (1) although one of the two marketplaces’ unsold returns are allowed, our contract is proven to reach the same goals as a complete return contract does if the two marketplaces face a same market demand uncertainty; (2) a feasible range for the buyback price, including a buyback price that the retailer can offset his unreturned inventory losses and a buyback price that the manufacturer can endure during the game, is presented for both members to negotiate; (5) our return contract provides both members a flexibility to determine which marketplace should implement the return practice when facing two different market demand uncertainties.

Table 2 further implicates a lower (higher) wholesale price in a lower (higher) market demand uncertainty. Table 2 offers a buyback commitment that only accepts one of the two marketplaces’ returns allows the retailer (manufacturer) to take less (more) risk of excess inventory. This complies with the real situation that the retailer (manufacturer) prefers to apply the return practice to a higher (lower) demand uner stand marketplace. Table 2 further implicates a lower (worse) than in lower (better) than in higher (lower) wholesale price in a lower (higher) market demand uncertainty. Table 2 always in favor of the retailer’s (manufacturer’s) profit. Taking as an example, we have (Better (worse) than ) at (, ) (Better (worse) than ) at (, ) This may be explained by the following managerial meanings. First, from the retailer’s standpoint, he is willing to play more order quantity in a less risky marketplace if the wholesale price is cheap. Second, from the manufacturer’s standpoint, he is willing to negotiate a cheap wholesale price in a high risky marketplace to entice the retailer’s more order.

IV. CONCLUSIONS

In this study, we investigated a one-leader (manufacturer)-one-follower (retailer) decentralized supply chain where the manufacturer provides a newsvendor-type item to a retailer who sells the item in two marketplaces. In order for the retailer to place an order up to the same level as in a centralized supply chain, not only is the manufacturer willing to lower the two wholesale prices compared to those in a price-only contract; the manufacturer also offers a buyback commitment that only accepts one of the two marketplaces’ returns for administration-cost-saving- and -different-demand-uncertainty concerns.

During the course of this study, the following are respectively resolved: (1) Our contract is proven to coordinate the decentralized supply chain; (2) we discover a range for the two wholesale prices that makes our contract a win-win situation; (3) although one of the two marketplaces’ unsold returns are allowed, our contract is proven to reach uncertainties. Thus, Table 2 compares the manufacturer’s standpoint if facing two different demand uncertainties. Table 2 compares

REFERENCES


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APPENDIX

Proof of Proposition 1

For \( i = 1, 2 \), we have

\[
\frac{\partial^2 E[\pi]_i}{\partial z_i^2} = (p_i - w_i + s_i) - (p_i + s_i)F(z_i), \quad \frac{\partial^2 E[\pi]_i}{\partial z_i}\partial z_i = -(p_i + s_i)f(z_i) \quad \text{and} \quad \frac{\partial^2 E[\pi]_i}{\partial z_i\partial z_i} = 0.
\]

Its Hess matrix is then given by \( H = \begin{bmatrix} -(p_i + s_i) & 0 \\ 0 & -(p_i + s_i)\end{bmatrix} \) which is negative-definite, and hence, induces the strict concavity of \( E[\pi]_i \) in \( z_i \). To prove the unique existence of the optimal \( z_i \), we solve the first-order necessary equations, resulting in

\[
z_i^* = F_i^{-1}\left(\frac{p_i - w_i + s_i}{p_i + s_i}\right)
\]

and

\[
Q_i^* = D_i + z_i^* = D_i + F_i^{-1}\left(\frac{p_i - w_i + s_i}{p_i + s_i}\right)
\]

for the price-only contract, and this completes the proof.

Proof of Proposition 2

For \( i = 1, 2 \), \( z_i = F_i^{-1}\left(\frac{p_i - w_i + s_i}{p_i + s_i}\right) \) implies \( F_i(z_i^*) = 1 - \frac{w_i}{p_i + s_i} \), from which if we differentiate both sides w.r.t \( w_i \),

\[
\frac{\partial^2 E[\pi]|_i}{\partial w_i^2} = -\frac{1}{w_iF(z_i^*)}
\]

is then obtained. Further, taking the first and second-order partial derivatives w.r.t \( w_i \) for \( E[\pi]|_i \) yields the following, respectively,

\[
\frac{\partial^2 E[\pi]|_i}{\partial w_i \partial z_i} = D_i + z_i^* - \frac{w_i - c}{w_iF(z_i^*)}, \quad \frac{\partial^2 E[\pi]|_i}{\partial z_i^2} = -\frac{w_i - c}{w_iF(z_i^*)}\left(\frac{w_i}{w_iF(z_i^*)}(z_i^*) + c(r^2(z_i^*) + 2c(r^2(z_i^*)))\right) \quad \text{and} \quad \frac{\partial^2 E[\pi]|_i}{\partial w_i \partial z_i} = 0,
\]

where the hazard rate \( r(z^*) = \frac{f(z^*)}{1 - F(z^*)} \). According to Barlow and Proschan [21], \( r(y) + r(x) > 0 \) holds for all non-decreasing hazard rate distributions, and this confirms \( \frac{\partial^2 E[\pi]|_i}{\partial z_i^2} < 0 \), making its corresponding Hessian matrix negative-definite and, thus, proving that \( E[\pi]|_i \) is strictly concave in \( w_i, i = 1, 2 \). The unique optimal

\[
\frac{z^*_i}{1 - (D_i + z_i^*)} = \frac{c}{1 - (D_i + z_i^*)}
\]

is then obtained by solving the first-order necessary conditions, and this completes the proof.